WEEKLY TEST MEDICAL PLUS -02 TEST - 09 Balliwala SOLUTION Date 01-09-2019

[PHYSICS]

1. Initial kinetic energy of mass

$$K_{\text{initial}} = \frac{1}{2} I\omega^2 = \frac{1}{2}(mr^2)\omega^2 = \frac{1}{2} m(r^2\omega^2)$$

$$= \frac{1}{2} mv_0^2$$

Conservation of angular momentum

$$mv_0R_0 = mv\frac{R_0}{2} \quad \Rightarrow \quad v = 2v_0$$

Final kinetic energy of mass

$$K_{\text{final}} = \frac{1}{2}mv^2 = \frac{1}{2}m(2v_0)^2 = 4.\frac{1}{2}mv_0^2 = 4k_{\text{initial}}$$

2. $\vec{L} = \vec{r} \times \vec{p}$

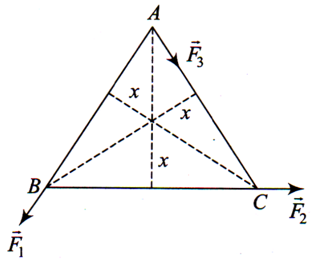
By right-hand screw, rule, the direction of $\vec{L}$ is perpendicular to the plane containing $\vec{r}$ & $\vec{p}$.

The mass is rotating in the plane, about a fixed point, thus this plane will contain $\vec{r}$ & $\vec{p}$ and the direction of $\vec{L}$ will be perpendicular to this plane.

3. If we take boat and both persons as a system, there is no external force acting on the system. The center of mass of the system is initially at rest and will be at rest as there is no external force acting on it to displace center of mass. Hence there is no shifting of center of mass.

4. Taking torque about O, net torque should be zero.

$$F_2 \times x - F_3 \times x + F_1 \times x = 0$$



$$F_3 = F_1 + F_2$$

5. From conservation of angular momentum

$$I\omega = mvr$$

$$200 \times \omega = 50 \times 2 \times 1$$

$$\omega = \frac{1}{2} \text{ rad/s}$$

$$v = r\omega = 1 \text{ m/s}$$

$$\therefore \quad T = \frac{2\pi r}{1-(-1)} = \frac{2\pi r}{2} = \pi r = 2\pi \text{ s}$$

6. Moment of inertia of circular disc = $\frac{1}{2}mR^2$. Thus, as the distance between the centre and the point increases, the moment of inertia increases.

7. $$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{300(0) + 500(40) + 400(70)}{300 + 500 + 400}$$

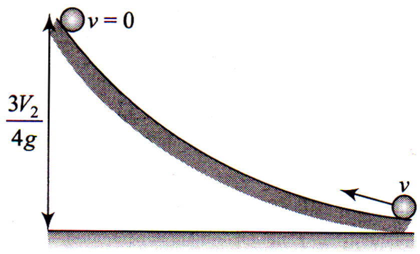
$$= \frac{20000 + 28000}{1200} = \frac{48000}{1200} = 40 \text{ cm}$$

8. $$\frac{1}{2} \times 3(4)^2 + \frac{1}{2} \times \frac{(3 \times R^2)}{2} \times \left(\frac{4}{R}\right)^2 = \frac{1}{2}Kx^2$$

$\Rightarrow \quad x = 0.6$ m

9. From law of conservation of mechanical energy

$$\Delta K + \Delta U = 0$$



$$\left[0 - \left(\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2\right)\right] + \left(mg \times \frac{3v^2}{4g}\right) = 0$$

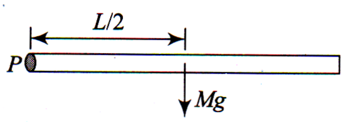
$$\Rightarrow \quad \frac{1}{2}I\omega^2 = \frac{3}{4}mv^2 - \frac{1}{2}mv^2 = \frac{mv^2}{2}\left(\frac{3}{2} - 1\right)$$

As cylinder is rolling $\omega = \frac{v}{R}$

or $\quad \frac{1}{2}I\frac{v^2}{R^2} = \frac{mv^2}{4} \quad$ or $\quad I = \frac{1}{2}mR^2$

Hence, object is a disc.

10. Taking torque about P



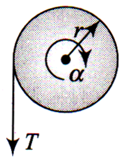
$$Mg\frac{L}{2}=\left(\frac{ML^2}{3}\right)\alpha$$

Hence $\alpha=\dfrac{3g}{2L}$

11. Applying torque equation about center of cylinder $\tau = I\alpha$

α is the angular acceleration of cylinder and it is given

$$\alpha = 2\ \text{revolution/s}^2 = 2\times 2\pi = 4\pi\ \text{rad/s}^2$$



$$Tr = I\alpha$$

$$T = \frac{I\alpha}{r}=\frac{mr^2}{2}\times\frac{\alpha}{r}=\frac{mr\alpha}{2}$$

$$= \frac{50\times 0.5\times 4\pi}{2}\ \text{N} = 157\ \text{N}$$

12. Acceleration of sphere when it is slipping down the incline , $a_{slipping} = g\sin\theta$

Acceleration of sphere when it is rolling down

$$a_{rolling}=\frac{g\sin\theta}{1+\dfrac{K^2}{r^2}}=\frac{5}{7}g\sin\theta$$

Hence required ratio $\dfrac{a_{rolling}}{a_{slipping}}=\dfrac{5}{7}$

13. Initial angular momentum $L_{initial} = mv_0R$

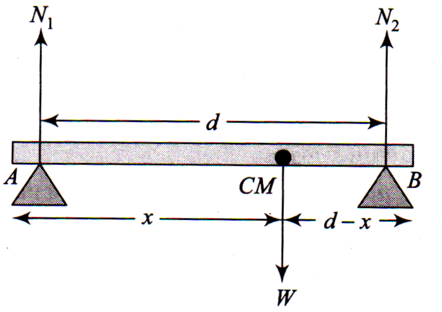
Initial angular momentum $L_{final} = mv\dfrac{R}{2}$

Conservation of angular momentum

$$mv_0R_0 = mv\frac{R_0}{2} \Rightarrow v = 2v_0$$

$$KE = \frac{1}{2}mv^2 = 2\,mv_0^2$$

14. Taking torque about end A

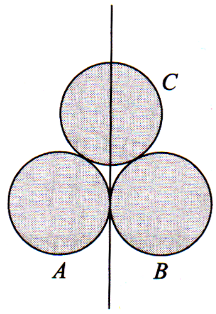


$$\tau_B = 0$$

$$\Rightarrow \quad N_1 d = W(d-x)$$

$$\Rightarrow \quad N_1 = \frac{W(d-x)}{d}$$

15. $I = I_A + I_B + I_C$



$$= \left(\frac{2}{3}mr^2 + mr^2\right) + \left(\frac{2}{3}mr^2 + mr^2\right) + \frac{2}{3}mr^2$$

$$= 4\,mr^2$$

16. Velocity of the automobile

$$v = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

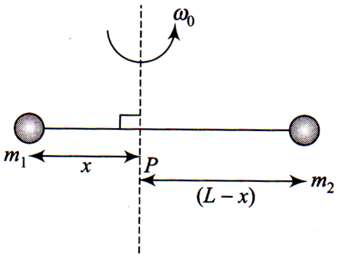
$$\omega_0 = \frac{v}{R} = \frac{15}{0.45} = \frac{100}{3} \text{ rad/s}$$

So angular acceleration

$$\alpha = \frac{\Delta\omega}{t} = \frac{\omega_f - \omega_0}{t} = -\frac{100}{45} \text{ rad/s}^2$$

So, torque $I\alpha = 3 \times \frac{100}{45} = 6.66 \text{ kg} - \text{m}^2\text{s}^{-2}$

17. The position of point P on rod through which the axis should pass so that the work required to set the rod rotating with minimum angular velocity ω_0 is their centre of mass



so $\quad m_1 x = m_2(L-x) \quad \Rightarrow \quad x = \dfrac{m_2 L}{m_1 + m_2}$

18. Angular momentum will be conserved if net **torque** actting on the system becomes zero.

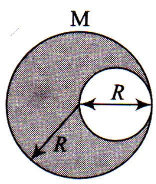
Given force acting $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$ **(i)**

and $\quad \vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k} = -2(-\hat{i} + 3\hat{j} + 6\hat{k})$ **(ii)**

If torque becomes zero then $\vec{r} \times \vec{F} = 0$

If $\alpha = -1$ then, $\vec{r} \times \vec{F} = 0$

19.



$$I_{\text{Total disc}} = \frac{MR^2}{2}$$

$$M_{\text{Removed}} = \frac{M}{d} \qquad \text{(Mass} \propto \text{area)}$$

I_{Removed} (about the same perpendicular axis)

$$= \frac{M}{4}\frac{(R/2)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2 = \frac{3MR^2}{32}$$

$$I_{\text{Removed disc}} = I_{\text{Total}} - I_{\text{Removed}}$$

$$= \frac{MR^2}{2} - \frac{3}{32}MR^2 = \frac{13}{32}MR^2$$

20. Particle at periphery will have both radial and tangential acceleration

$$a_t = R\alpha = 0.5 \times 2 = 1 \text{ m/s}^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = 0 + 2 \times 2 = 4 \text{ rad/sec}$$

$$a_c = \omega^2 R = (4)^2 \times 0.5 = 16 \times 0.5 = 8 \text{ m/s}^2$$

$$a_{\text{total}} = \sqrt{a_p^2 + a_c^2} = \sqrt{1^2 - 8^2} \approx 8 \text{ m/s}^2$$

21. $$a = \frac{g \sin\theta}{1 + \frac{K^2}{R^2}}$$

For disc $\frac{K^2}{R^2} = \frac{1}{2} = 0.5$

For sphere $\frac{K^2}{R^2} = \frac{2}{5} = 0.4$

$a(\text{sphere}) > a(\text{disc})$

$\therefore$ sphere reaches first

22. $v = 36$ km/h $= 10$ m/s

By law of conservation of momentum

$$2 \times 10 = (2+3)V \quad \Rightarrow V = 4 \text{ m/s}$$

Loss in K.E. $= \frac{1}{2} \times 2 \times (10)^2 - \frac{1}{2} \times 5 \times (4)^2$

$= 60$ J

From law of conservation of momentum, we have

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$\Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$$

Given,

$$m = 2 \text{ kg}, \; u_1 = 36 \times \frac{5}{18} = 10 \text{ m/s},$$

$$m_2 = 3 \text{ kg}, \; u_2 = 0$$

$$\therefore \quad v = \frac{2 \times 10 + 3 \times 0}{2+3} = 4 \text{ m/s}$$

Loss in kinetic energy is

$$\Delta K = \Delta K' - \Delta K''$$

$$= \left\{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right\} - \left\{\frac{1}{2}(m_1 + m_2)v^2\right\}$$

$$= \frac{1}{2} \times 2 \times (10)^2 - \frac{1}{2} \times 5 \times (4)^2$$

$$= 100 - 40 = 60 \text{ J}$$

23. The moment of inertia (I) of a body about an axis is equal to the torque (τ) required to produce unit angular acceleration (α) in the body about that axis is

$$\tau = I\alpha$$

Given, $\alpha = 4\pi$ rad/s^2, $\tau = 31.4$ Nm

$$\Rightarrow \quad I = \frac{\tau}{\alpha} = \frac{31.4}{4 \times 3.14} = 2.5 \text{ kg-m}^2$$

24. $\frac{2}{3}MR_h^2 = \frac{2}{5}MR_S^2$ or $\frac{R_h^2}{R_S^2} = \frac{3}{5}$ or $\frac{R_h}{R_s} = \sqrt{\frac{3}{5}}$

25. Angular velocity is the vector quantity which represents the process of rotation (change of orientation) that occurs at as instant of time. For a rigid body it supplements translational velocity of the centre of mass to describe the full motion. The line of direction of the angular velocity is given by the axis of rotation and the right-hand rule indicates the direction.

26. If no external force acts upon a system of two (or more) bodies then the total momentum of the system remains constant.

Hence, momentum before collision = momentum after collision

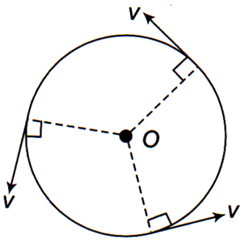
$m_1u_1 = m_2v_2$

Given, $u_1 = 1$ m/s, $m_2 = 0.05$ kg, $v_2 = 30$ m/s

$\Rightarrow \quad m_1 \times 1 = 0.05 \times 30$

$\Rightarrow \quad m_1 = 1.5$ kg

27. If a body is rotating about an axis, then the sum of the moments of the linear moment of all the particles about the given axis is called the angular momentum of the body about that axis.



$J = I\omega = mrv$

Since direction of velocity is perpendicular to orbital plane and $J \propto v$, therefore in an orbital motion the angular momentum vector is perpendicular to the orbital plane.

28. Fractional energy decreases in kinetic energy of neutron

$$= 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \qquad [\text{As } m_1 = 1 \text{ and } m_2 = 2]$$

$$= 1 - \left(\frac{1-2}{1+2}\right)^2 = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

29. K.E. of ball in position $B = mg(R - r)$

Here m = mass of ball.

Since it rolls without slipping the ratio of rotational to translational kinetic energy will be 2/5.

$$\frac{K_R}{K_T} = \frac{2}{5}$$

$$K_T = \frac{2}{7} mg(R - r)$$

$$\frac{1}{2} mv^2 = \frac{2}{7} mg(R - r)$$

$$v = \frac{2}{\sqrt{7}} \sqrt{g(R - r)}$$

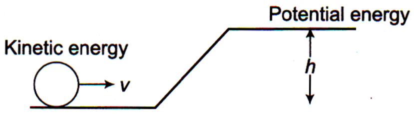
$$\omega = \frac{v}{R - r} = 2\sqrt{\frac{g}{7(R - r)}}$$

30. From law of conservation of angular momentum if no external torque is acting upon a body rotating about an axis, then the angular momentum of the body remains constant that is

$$J = I\omega = \text{constant}$$

Hence, if decreases, ω increases and vice–versa. When liquid is dropped, mass increases hence I increases ($I = mr^2$). So, ω decreases, but as soon as the liquid starts falling ω increases again.

31. From law of conservation of energy, energy can neither be created nor be destroyed but it remains conserved. In the given case the sum of kinetic energy of rotation and translation is converted to potential energy.



Also moment of inertia of disc is

$$I = \frac{2}{5} MR^2$$

$$\therefore \quad \underset{\text{(Translational kinetic energy)}}{\frac{1}{2} mv^2} + \underset{\text{(Rotational energy)}}{\frac{1}{2} I\omega^2} = \underset{\text{(Potential energy)}}{mgh}$$

$$\Rightarrow \quad \frac{1}{2} mv^2 + \frac{1}{2}\left(\frac{2}{5} MR^2\right)\frac{v^2}{R^2} = mgh$$

where $v = R\omega$, $\omega =$ angular velocity

$$\Rightarrow \quad \frac{7}{10} mv^2 = mgh$$

$$\Rightarrow \quad v = \sqrt{\frac{10}{7} gh}$$

Hence, to climb the inclined surface the velocity should be greater than $\sqrt{\frac{10}{7} gh}$.

32. Change in momentum = Impulse

= Area under force–time graph

$\therefore$ mv = Area of trapezium

$$\Rightarrow \quad mv = \frac{1}{2}\left(T + \frac{T}{2}\right)F_0$$

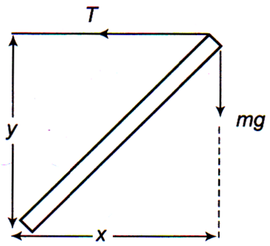
$$\Rightarrow \quad mv = \frac{3T}{4}F_0 \quad \Rightarrow \quad F_0 = \frac{4mu}{3T}$$

33. For equilibrium of street light,

$$mg \times x = T \times y$$

or $$T = \frac{mg.x}{y}$$

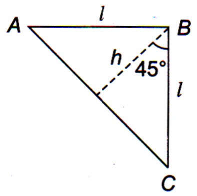
For T to be minimum, y should be maximum. Hence, pattern A is more sturdy.



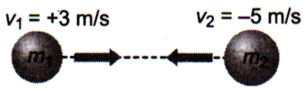
34.

$$h = l\cos 45° = \frac{l}{\sqrt{2}}$$

$$I_{AC} = \frac{1}{6}Mh^2 = \frac{1}{6}M\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{Ml^2}{12}$$



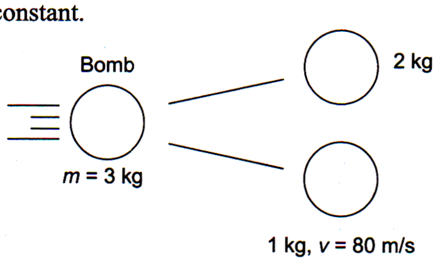
35. As $m_1 = m_2$, therefore after elastic collision velocities of masses get interchanged



i.e., velocity of mass $m_1 = -5$ m/s

and velocity of mass $m_2 = +3$ m/s

36. From law of conservation of momentum, when no external force acts upon a system of two (or more) bodies then the total momentum of the system remains constant.



Momentum before explosion = momentum after explosion

Since bomb v at rest, its velocity is zero, hence

$$mv = m_1v_1 + m_2v_2$$

$$3 \times 0 = 2v_1 + 1 \times 80$$

$$\Rightarrow \quad v_1 = -\frac{80}{2} = -40 \text{ m/s}$$

Total energy imparted is

$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2} \times 2 \times (-40)^2 + \frac{1}{2} \times 1 \times (80)^2$$

$$= 1600 + 3200 = 4800 \text{ J}$$

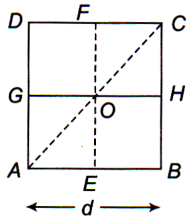
$$= 4.8 \text{ kJ}$$

37. Let the each side of square lamina is d.

So, $I_{EF} = I_{GH}$ (due to symmetry)

and $I_{AC} = I_{BD}$ (due to symmetry)

Now, according to theorem of perpendicular axis,



$$I_{AC} + I_{BD} = I_0$$

$$\Rightarrow \quad 2I_{AC} = I_0 \quad \ldots\text{(i)}$$

$$\text{and} \quad I_{EF} + I_{GH} = I_0$$

$$\Rightarrow \quad 2I_{EF} = I_0 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get $I_{AC} = I_{EF}$

$$\therefore \quad I_{AD} = I_{EF} + \frac{md^2}{4}$$

$$= \frac{md^2}{12} + \frac{md^2}{4} \quad \left(\text{as } I_{EF} = \frac{md^2}{12}\right)$$

So, $I_{AD} = \dfrac{md^2}{3} = 4I_{EF}$

38. In an inelastic collision, the particles do not regain their shape and size completely after collision. Thus, the kinetic energy of particles no longer remains conserved. However, in the absence of external forces, law of conservation of linear momentum still holds good.

39. 

Initial momentum = mv

Final momentum = $3mV$

By the law of conservation of momentum

$$mv = 3mV$$

$\therefore \quad V = v/3$

40. We have $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ ...(ii)

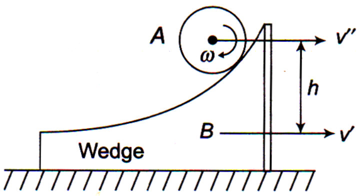
Given, $\alpha = 3.0$ rad/s^2, $\omega_0 = 2.0$ rad/s, $t = 2$s

Hence, $\theta = 2 \times 2 + \frac{1}{2} \times 3 \times (2)^2$

or $\quad \theta = 4 + 6 = 10$ rad

41. Due to elastic collision of bodies having equal mass, their velocities get interchanged.

42. At the maximum height vertical velocity of cylinder is zero, but horizontal velocity of the wedge and cylinder will be same.



In the absence of friction between the cylinder and the wedge surface, angular velocity of cylinder remains constant. From energy conservation:

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv'^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv'^2 + mgh \quad ..(i)$$

By the conservation of linear momentum

$$mv = 2mv' \quad \Rightarrow v' = v/2 \quad \dots (ii)$$

From (i) and (ii),

$$h = v^2/4g$$

43. When ball falls vertically downward from height h_1 its velocity $\vec{v}_1 = \sqrt{2gh_1}$

and its velocity after collision $\vec{v}_2 = \sqrt{2gh_2}$

Change in momentum

$$\Delta\vec{P} = m(\vec{v}_2 - \vec{v}_1) = m(\sqrt{2gh_1} + \sqrt{2gh_2})$$

(because $\vec{v}_1$ and $\vec{v}_2$ are opposite in direction)

44. In both the cases, the loss of gravitational potential energy and the resulting gain of 'total kinetic energy' is same.

45. Area between curve and displacement axis

$$= \frac{1}{2}\times(12+4)\times10 = 80 \text{ J}$$

In this time the body acquires kinetic energy $= \frac{1}{2}mv^2$ by the law of conservation of energy

$$\frac{1}{2}mv^2 = 80 \text{ J}$$

$\Rightarrow \quad \frac{1}{2}\times0.1\times v^2 = 80$

$\Rightarrow \quad v^2 = 1600$

$\Rightarrow \quad v = 40$ m/s